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TITLE- A Study of Methods of Augmenting Cross Product Steering with Direct Control of Out-of-Plane Position Errors - Description of the Methods Used

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ABSTRACT

This memorandum is the first of a series of four which discusses the relative merits of augmenting the cross product steering law with direct control of the out-of-plane position error. This memorandum discusses the methods used in the studies and develops the steering equations tested. The subsequent memorandums will discuss actual results of the studies for the Lunar Orbit Insertion, Transearth Injection and Translunar Injection powered flight phases.

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SUBJECT: A Study of Methods of Augmenting Cross Product Steering with Direct Control of Out-of-Plane Position Errors - Description of the Methods Used - Case 310

DATE: November 25, 1966

FROM: D. A. Corey

TM-66-2012-6

TECHNICAL MEMORANDUM

INTRODUCTION

This memorandum is the first of four which will discuss the results of studies concerning the relative merits of augmenting the cross product steering law with direct control of the out-of-plane errors. This first memorandum of the series discusses the development of the steering equations studied and, to some extent, the simulation used to perform the study. The remaining three memorandums will discuss the achieved results of the study for the Lunar Orbit Insertion (LOI), Transearth Injection (TEI) and Translunar Injection (TLI) (with cross product steering) powered flight maneuvers.

A study of out-of-plane steering for the lunar orbit insertion maneuver has previously been conducted by A. W. Merz of MIT/IL (reference 1). Similarities and differences between the approaches to the problem will be noted when applicable.

Accurate control of the achieved orbit plane is probably most important for the LOI maneuver since any error in achieving the desired plane would result in forcing the Lunar Module to make a plane change during powered descent if it is to achieve its desired landing site. Similarly, the magnitude of the plane change required of the CSM prior to LM ascent or of the LM itself during its ascent is directly controlled by the accuracy of the LOI maneuver with respect to the achieved plane. Whereas, out-of-plane errors resulting from TEI and TLI maneuvers can be corrected in the midcourse correction phases with a small ΔV penalty, it is nevertheless of interest to determine whether more accurate steering can produce a meaningful reduction in that ΔV penalty.

DESCRIPTION OF THE SIMULATION

The studies were conducted using the Monte Carlo II Powered Flight Error Analysis Simulator. A detailed description of the computer program is provided in forthcoming documentation (reference 2). Consequently, only a brief description will be presented here.

The Monte Carlo II program simulates primary engine powered flights. The simulator was principally designed to develop powered flight transition matrices which relate deviations and uncertainties in the state vector at the end of the powered flight phase to deviations and uncertainties in the state vector at the beginning of the phase and to vehicle performance and sensing errors which occur during the phase. For the purposes of these memoranda deviations, or actual deviations, refer to the difference between the actual state vector and the reference trajectory state vector, and uncertainties refer to differences between the actual state vector and the estimated state vector. All three types of perturbations are developed at prespecified reference times.

The simulation included a coasting or free flight phase before and after the powered flight maneuver. The duration of these coasting intervals is 10 seconds for the nominal (no error) trajectory. Thus the simulation starts 10 seconds prior to nominal engine ignition and ends 10 seconds after nominal engine cutoff. The transition matrices then relate perturbations existing at a time 10 seconds past nominal engine cutoff to perturbations which were present 10 seconds before nominal engine ignition and to errors which occur during the powered portion of simulation. The transition matrices may then be used to propagate the covariance matrices of actual deviations and of uncertainties which exist before the phase, through the phase, obtaining the actual and the uncertainty covariance matrices at the end of the phase.

The covariance matrices discussed in these memoranda will all be in the orbit plane or UVW coordinate system. The radial or U component is in the direction of the nominal position vector. The out-of-plane or W component is defined by crossing the nominal position vector $\underline{P}$ with the desired velocity vector $(\underline{V}_{DES})$. Thus $\underline{i}_W = \text{UNIT}\ (\underline{P} \times \underline{V}_{DES})$. The downrange or V component completes the right-handed orthogonal coordinate system and is in the general direction of the velocity vector.

The simulator also determines the values of several parameters which describe the initial and final conics for both nominal and off nominal cases. In addition, the computed time-to-go and the magnitude of the velocity to be gained at the time of engine cutoff are computed and printed. These provide indications of the ability of the guidance scheme to achieve the desired conditions based on the estimated state vector of the vehicle. Ideally, they should both be zero.

A simplified representation of an auto pilot is included in the simulation to account for the effects and restrictions of vehicle dynamics. The commanded pitch and yaw gimbal angle changes are restricted to a maximum value of 0.1 radian per second. The maximum allowable pitch and yaw angular accelerations are inversely proportional to the vehicle mass and are based on externally computed values which are input items. These values are computed at the beginning and the end of the flight and are based on the vehicle mass, center of gravity, moments of inertia, and maximum possible engine gimbal angle. The angular acceleration limitation is assumed identical for both pitch and yaw. The commanded roll angle was held constant at zero.

GUIDANCE EQUATIONS

Generally, the current concept of guidance for the CSM is to use cross product steering. Since this steering law has been treated extensively in many documents (e.g., reference 3) a brief description will suffice here.

A required velocity, $\underline{V}_r$, is computed which, if achieved instantaneously, would satisfy the objectives of the particular powered flight phase.

The instantaneous velocity to be gained is then

$$\underline{V}_g = \underline{V}_r - \underline{V} \qquad (1)$$

where $\underline{V}$ is the present velocity of the vehicle.

The steering principle is then to point the thrust vector such that

$$\underline{a}_T \times \underline{V}_g = c\underline{b} \times \underline{V}_g \qquad (2)$$

where:

$\underline{a}_T$ is the acceleration due to thrust

$$\underline{b} = \frac{d\underline{V}_r}{dt} - \underline{g} \qquad (3)$$

$\underline{g}$ is the acceleration due to gravity and

c is a scalar factor generally taking a value between 0 and 1.

Additional manipulation yields a solution for the direction of the desired acceleration vector $\underline{i}_T$

$$\underline{i}_T = \text{UNIT}\left(c\underline{b} + (q - \underline{i}_g \cdot c\underline{b})\underline{i}_g\right) \tag{4}$$

where:

$$q = \left(a_T^{\,2} - |c\underline{b}|^2 + (\underline{i}_g \cdot c\underline{b})^2\right)^{1/2} \quad , \tag{5}$$

$$\underline{i}_g = \frac{\underline{V}_g}{|\underline{V}_g|} \quad , \tag{6}$$

and

a_T = the available thrust acceleration magnitude.

The quantity $\underline{V}_r$ varies with the particular phase. The applicable expressions will be included with the results for the particular phase.

With considerable algebraic manipulation it is possible to derive an analytic expression for $\underline{b}$. However, a completely satisfactory approximation of $\underline{b}$ may be obtained by simply obtaining the change in $\underline{V}_r$ between two computation intervals. This latter method was used in this study. Thus,

$$\underline{b} = \frac{\underline{V}_r(t) - \underline{V}_r(t-\Delta t)}{\Delta t} + \frac{\mu}{|\underline{r}(t)|^3}\,\underline{r}(t) \tag{7}$$

where

$\underline{r}(t)$ is the present vector

and

Δt is the time interval between computations.

DEVELOPMENT OF OUT-OF-PLANE STEERING LAWS

Note that the cross product steering concept provides no direct control of the final position of the vehicle. Only the final velocity is controlled directly. As a consequence, the vehicle must be on the nominal trajectory before the maneuver, and engine ignition time and performance must be nominal in order for the vehicle to achieve the desired orbit precisely. In terms of the out-of-plane control, the cross product steering scheme attempts to put the vehicle on a plane parallel to the desired plane in the sense that the final velocity is parallel to the desired plane (neglecting uncertainty errors) but no control of the out-of-plane position error is provided.

Actually, it is not the cross product law itself which produces these effects, rather, it is the selection of the required velocity $\underline{V}_r$. Two possibilities suggest themselves for control of out-of-plane position. One is to define the $\underline{V}_r$ vector in such a way as to implicitly control out-of-plane position (or other components of position). Some possibilities of this type of approach are currently under study. A second possibility is to use cross product steering, with the $\underline{V}_r$ vector defined as presently planned, to control the in-plane components of the thrust acceleration vector and develop another law to control the out-of-plane component. The latter method is considered in this memorandum. For the sake of brevity (forsaking complete accuracy), this memorandum will refer to the out-of-plane control as yaw-steering.

DEVELOPMENT OF A LINEAR YAW-STEERING LAW

Since it is necessary to control both the final out-of-plane position and velocity, at least two parameters or variables are required in the formulation. Let a_n be the desired acceleration in the out-of-plane direction. That is, a_n will be perpendicular to the desired orbital plane. Then let

$$a_n = a + b(T-t)$$

where

T is the time from engine ignition to engine cutoff

t is the current time since engine ignition

a and b are parameters to be computed.

Integrating the first time

$$\int_t^T a_n(\tau)d\tau = \int_t^T (a+b(T-\tau))d\tau$$

or

$$\dot{X}(T) - \dot{X}(t) = \int_t^T (a+b(T-\tau))d\tau$$

making the change of variable

$$s = T - \tau$$

$$\dot{X}(T) - \dot{X}(t) = \int_0^{T-t} (a+bs)ds$$

$$= a(T-t) + \frac{1}{2} b (T-t)^2$$

$\dot{X}(T)$ is the terminal velocity

$\dot{X}(t)$ is the present velocity.

Since $\dot{X}(T)$ is fixed, a second integration yields the following:

$$\int_t^T (X(T) - X(\tau))d\tau = X(T)(T-t) - (X(T) - X(t))$$

$$= \int_t^T (a(T-t) + \frac{1}{2}\, b(T-\tau)^2)d\tau$$

making the same change of variables yields

$$\dot{X}(T)(T-t) - (X(T) - X(t)) = \int_0^{T-t} (as + \frac{1}{2}\, bs^2)ds$$

$$= \frac{1}{2}\, a(T-t)^2 + \frac{1}{6}\, b(T-t)^3$$

Some additional manipulation yields the expressions for a and b.

$$a = \frac{4\dot{X}(T) + 2\dot{X}(t)}{(T-t)} - \frac{6(X(T) - X(t))}{(T-t)^2}$$

$$b = \frac{12(X(T) - X(t))}{(T-t)^3} - \frac{6(\dot{X}(T) + \dot{X}(t))}{(T-t)^2}$$

Since $\dot{X}(T)$ and $X(T)$ are to be driven to zero, further simplification yeilds

$$a = \frac{2\dot{X}(t)}{(T-t)} + \frac{6X(t)}{(T-t)^2}$$

$$b = -\frac{12X(t)}{(T-t)^3} - \frac{6\dot{X}(t)}{(T-t)^2}$$

$X(t)$ and $\dot{X}(t)$ are the present position and velocity deviations from the desired orbital plane which, for the sake of conformity with Merz, will be called TPD and TVD respectively. They are calculated by

$$TPD = \underline{r} \cdot \underline{i}_n \tag{9}$$

and

$$TVD = \underline{v} \cdot \underline{i}_n \tag{10}$$

where

$\underline{i}_n$ is a unit normal to the desired plane -- in the direction of the angular momentum vector.

Now, (T-t) is the estimated time until engine cutoff, call it Tgo.

Thus, a and b reduce to

$$a = \frac{2\ TVD}{Tgo} + \frac{6\ TPD}{Tgo^2} \tag{11}$$

$$b = -\frac{6\ TVD}{Tgo^2} - \frac{12\ TPD}{Tgo^3} \tag{12}$$

and

$$a_n = a + b\ Tgo \tag{13}$$

The commanded out-of-plane engine acceleration, TAC, can include a term to compensate for gravity

$$TAC = a + b\ Tgo + \frac{\mu\ TPD}{|\underline{r}|^3} \tag{14}$$

The gravity term is generally small and disappears as TPD goes to zero so it can be left out.

The total thrust acceleration commanded is then determined by allowing the out-of-plane acceleration to take on the full value, TAC, provided, of course, TAC does not excede the total available acceleration a_T. The commanded value of in-plane thrust acceleration, a_{ip}, is given the magnitude

$$a_{ip} = (a_T^{\ 2} - TAC^2)^{1/2} \tag{15}$$

The commanded in-plane acceleration vector, $\underline{i}_{ip}$, is computed by first computing $\underline{i}_T$ using the cross product steering law, (equation 4). $\underline{i}_T$ will generally have a component out of the desired plane as it attempts to null the out-of-plane velocity error. Only the in-plane components are used. Thus

$$\underline{i}_{ip} = \text{UNIT}\ (\underline{i}_T - (\underline{i}_T \cdot \underline{i}_n)\underline{i}_n) \tag{16}$$

The final commanded acceleration, $\underline{a}_c$ is then given by

$$\underline{a}_c = a_{ip}\ \underline{i}_{ip} + TAC\ \underline{i}_n \tag{17}$$

UNIT ($\underline{a}_c$) is then used to determine the desired platform gimbal angles.

The equation used for the out-of-plane acceleration has previously been reported by G. W. Cherry (reference 4). As Cherry notes, the quantities a and b are theoretically constants. However, in practice, computational round off errors, inability of the vehicle to respond perfectly, and the fact that the quantity Tgo is not a perfect estimator, cause these quantities to vary to some extent. For purposes of this study, they were simply recomputed during each cycle through the guidance equations. Furthermore, as Tgo gets small, the ability of the vehicle to correct out-of-plane position errors diminishes. As a consequence, during the last few seconds of flight, the computed values of TAC tend to diverge, and dominate the commanded acceleration computation. This has the effect of degrading the final orbit and is wasteful of fuel. Cherry suggests dropping the final out-of-plane position constraint during the last few seconds of flight and computing the out-of-plane acceleration solely on the basis of the out-of-plane velocity deviation. It was found, however, that dropping the out-of-plane steering altogether and relying solely on cross product steering yielded smaller out-of-plane velocity errors. Undoubtedly this is due to the direct dependence of the Tgo calculation on the cross product steering computations. Recall that the cross product steering law does attempt to null out-of-plane velocity errors.

A detailed study of the optimum time to cut off the out-of-plane steering was not conducted. Three values were tried, Tgo < 6, Tgo < 10, and Tgo < 15. Tgo < 6 produced the smallest cut off errors and so was used in the remainder of the studies.

An accurate estimate of Tgo is required in order to implement the equation for out-of-plane acceleration. The Tgo estimator developed by E. M. Copps (Reference 5) was found to produce excellent results and was used in the studies. Tgo is computed on the basis of the quantities developed for the cross product or in-plane steering. The derivation of the Tgo computation assumes that the constant c = 1. However, it was found to work equally well for c = 0, and c = 0.5.

$$\mathrm{Tgo} = \frac{1}{\lambda_1 - \lambda_2} \log \left(\frac{\lambda_2 (\lambda_1 + \omega)}{\lambda_1 (\lambda_2 + \omega)} \right) \tag{18}$$

where:

$$\omega = \frac{K_2^{\ 2} \, |\underline{V}_g|}{2 \, a_T}$$

$$\lambda_1 = \frac{1}{2\tau} + \frac{1}{2} K_1 + \frac{1}{2} \left(K_1^{\ 2} + 2K_2^{\ 2} + \frac{1}{\tau^2} - \frac{2}{\tau} K_1 \right)^{1/2}$$

$$\lambda_2 = -\frac{1}{2\tau} + \frac{1}{2} K_1 - \frac{1}{2} \left(K_1^{\ 2} + 2K_2^{\ 2} + \frac{1}{\tau^2} - \frac{2}{\tau} K_1 \right)^{1/2}$$

$$K_1 = \frac{\underline{cb} \cdot \underline{V}_g}{|\underline{V}_g|^2}$$

$$K_2 = \left(\frac{|\underline{cb}|^2}{|\underline{V}_g|^2} - K_1^2\right)^{1/2}$$

and

$$\frac{1}{\tau} = \frac{g_o \; I_{sp}}{a_T}$$

The value of Tgo was computed during each cycle through the guidance equations and was used to determine the time of engine cut off. Consequently the TAC equation can actually be simplified to:

$$TAC = -\frac{4\ TVD}{Tgo} - \frac{6\ TPD}{Tgo^2} + \frac{\mu\ TPD}{|r|^3} \tag{19}$$

As Tgo and $|\underline{V}_g|$ approach zero, the steering equations become somewhat eratic (even disregarding the effect of TAC noted previously) due to numerical round off errors and the limited ability of the vehicle to respond to rapidly changing gimbal angle commands. Consequently, once Tgo falls below 2 seconds, the previously commanded thrust vector orientation is held for the remainder of the powered flight. Furthermore, the current value of Tgo together with the previous value of Tgo are linearly extrapolated to determine the commanded engine cutoff time. This method has been found to yield a value of Tgo at cutoff of less than .05 seconds when the cycle interval through the guidance equations is two seconds. It should be mentioned that the simulation did not consider the effects of accelerometer quantizations or the effects of vibration. The cutoff errors would undoubtedly be larger if these effects were considered.

One of the problems encountered in installing yaw-steering in the command module steering equations is the shortage of available core storage in the AGC (Apollo Guidance Computer).

Copps' time-to-go equation, while quite accurate, is rather bulky. Consequently, a simpler, though less accurate, Tgo calculation was also tried, namely:

$$Tgo = |\underline{V}_g| \ / \ |\dot{\underline{V}}_g| \tag{20}$$

where $\underline{V}_g$ is as computed in equation 1 and

$$\dot{\underline{V}}_g = \underline{b} - a_T \, \underline{i}_T$$

b is as computed in equation 7

$\underline{i}_T$ is as computed in equation 4.

A comparison of the relative accuracy of the two computations is presented in Figure 1.

Development of a Quadratic Yaw-Steering Law:

As Cherry also pointed out in Reference 4, $a_n = a + b$ Tgo is not necessarily an optimum steering law. Another was also tried;

$$a_n = c + d \, Tgo + e \, Tgo^2 \tag{21}$$

This equation allows the constraint of the final value of the out-of-plane acceleration as well as position and velocity. A value of zero final acceleration was chosen. It may be shown that the coefficients c, d, and e are determined by

$$c = 0 \tag{22}$$

$$d = \frac{6 \ TVD}{Tgo^2} + \frac{24 \ TPD}{Tgo^3} \tag{23}$$

$$e = -\frac{12 \ TVD}{Tgo^3} - \frac{36 \ TPD}{Tgo^4} \tag{24}$$

When TPD and TVD are recomputed each cycle through the guidance equations the TAC computation reduces to:

$$TAC = -\frac{6\ TVD}{Tgo} - \frac{12\ TPD}{Tgo^2} + \frac{\mu\ TPD}{|r|^3} \qquad (25)$$

Comparison with Merz's Method:

Merz's paper (reference 1) presents the equation

$$TAC = \left[-\frac{6\ TPD}{Tgo^2} - \frac{4\ TVD}{Tgo}\right] + \left[\frac{12\ TPD}{Tgo^3} + \frac{6\ TVD}{Tgo^2}\right](T-t) \qquad (26)$$

where T is the predicted time of engine cutoff and t is the time since the previous calculation of TPD and TVD.

There is probably a typographical error since it can be shown that the coefficients used in equation 26 are the solution for the boundary value problem

$$\ddot{X} = a + bt \qquad (27)$$

rather than

$$\ddot{X} = a + b(T-t)$$

Note that if T - t = 0 in equation 26 (or actually t = 0 in equation 27) the value of TAC is identical to that computed by equation 19, (neglecting the gravity term). Using Δt, the interval between computations, in place of (T-t) in equation 25 was tried and found to yield essentially identical results to those produced by using equation 19 as expected.

Merz used only the components of the position vector which lie in the desired plane in his calculation of $\underline{V}_r$. He defines a "decremented" position vector $\underline{P}_d$ such that

$$\underline{P}_d = \underline{r} - (\underline{r} \cdot \underline{i}_n)\underline{i}_n \tag{28}$$

$\underline{P}_d$ was then used to compute $\underline{V}_r$. Thus $\underline{V}_r$ lies in the desired plane but note that

$$\underline{V}_g = \underline{V}_r - \underline{V}$$

will still have an out-of-plane component. The thrust acceleration vector $\underline{i}_T$ computed using the cross product steering law will also have an out-of-plane component. Consequently, this is not the same as the operation on $\underline{i}_T$ performed in equation 16. These studies included runs with and without the use of a decremented position vector for computing $\underline{V}_r$.

Merz' equations for commanding the pitch and yaw gimbals assume a simplification that was not made in these studies. He effectively uses cross product steering alone to command the pitch gimbal, and the value of TAC alone to command the yaw gimbal. The pitch plane, however, is never aligned in the final orbit plane except when the maneuver involves no plane change. As a consequence, the cross product steering law is to some extent controlling the out-of-plane acceleration and the yaw-steering law is to some extent controlling the in-plane component of acceleration. This cross coupling probably caused some of the comparatively large cutoff errors as well as the comparatively large gimbal angle excursions Merz reported.

CONCLUSIONS

Two different equations for providing direct control of out-of-plane position errors have been developed. Subsequent papers will discuss the effectiveness of the two methods as compared to the effectiveness of cross product steering alone. The papers will also include results concerning

1. Use of the gravity term in the out-of-plane acceleration command.

2. Use of the decremented position vector in the computation of $\underline{V}_r$.

3. The relative effects of an accurate computation of Tgo vs a less accurate but simpler computation.

The quantities of interest in these comparisons will be the amount of fuel required for each scheme, the sensitivity of the schemes to various types of errors, and the comparitive magnitudes of the resulting errors, both in-plane and out-of-plane for cases which bound the family of Apollo missions.

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2012-DAC-jdc

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Attachments
References
Figure I

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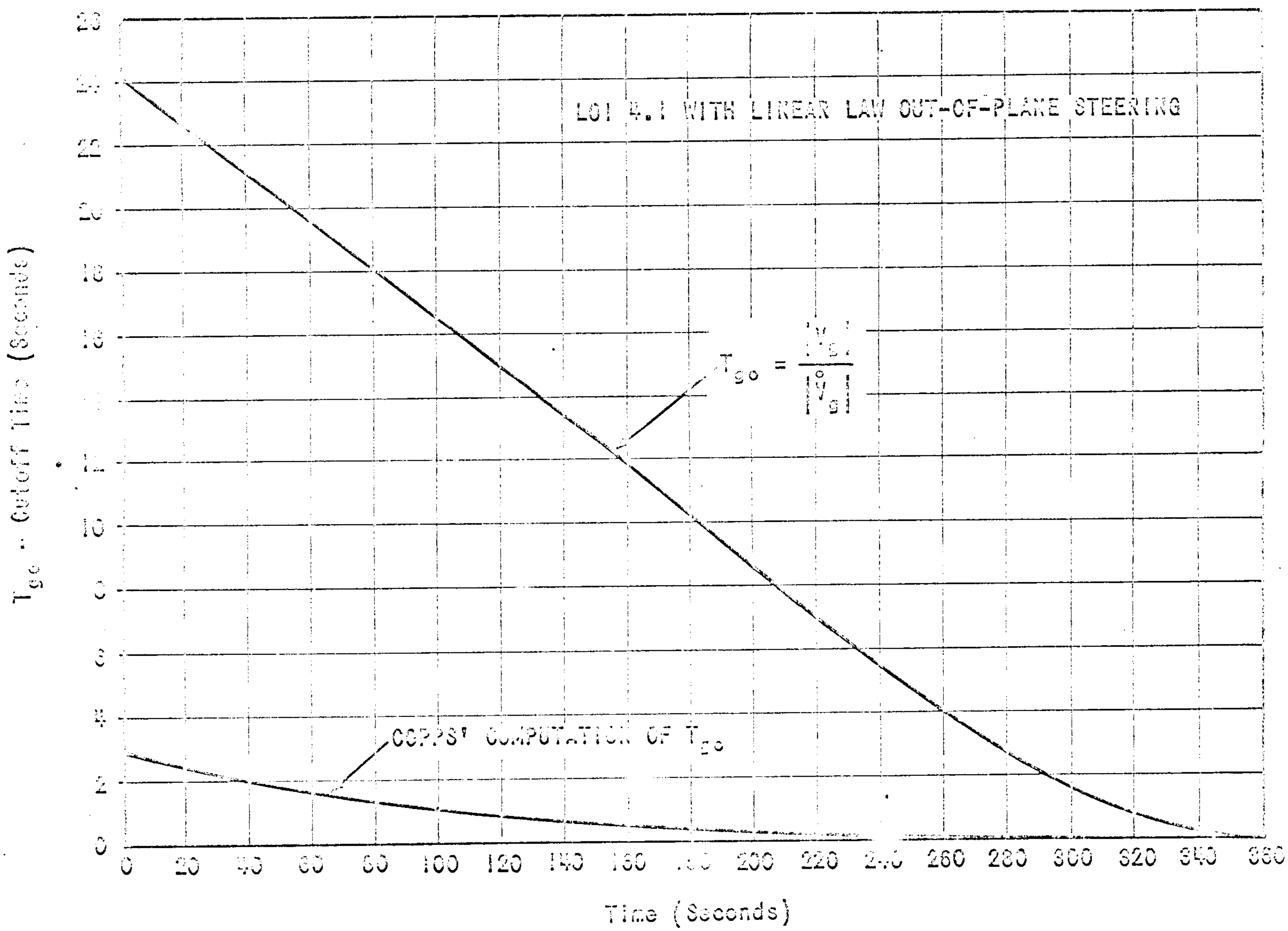

FIGURE 1 - TIME-TO-GO ERROR VS. TIME